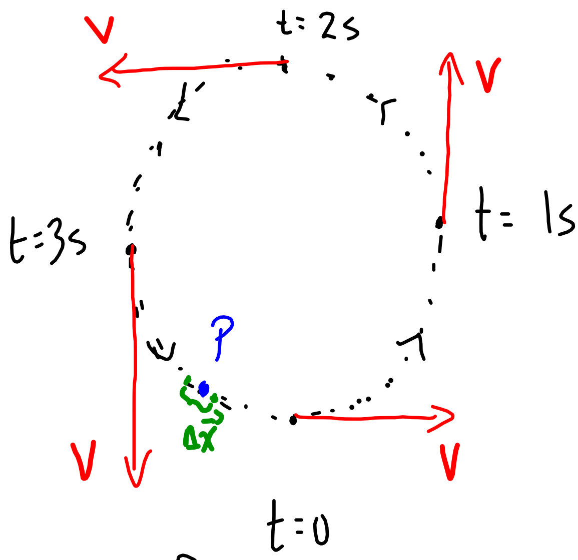
Instantaneous Velocity



The instantaneous velocity of the object at a point P is determined by measuring the displacement $\Delta\vec{x}$ over a small time interval when it passes P.

$$\vec{V}_{inst} = \frac{\Delta\vec{x}}{\Delta t}$$ when Δt is very small (almost zero)

$$\vec{V}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\vec{x}}{\Delta t}$$

The instantaneous velocity of an object at a point P is its velocity at the instant that it passes P and it is the average velocity of the object calculated over an infinitely small (but not zero) displacement around P.

Acceleration

We have already defined acceleration:

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

A better way (more accurate) is to define acceleration as the rate of change of instantaneous velocity

$$\vec{a} = \frac{\Delta \vec{v}_{inst}}{\Delta t}$$

Acceleration can be the average acceleration in a range or the instantaneous acceleration at a point

average acceleration: $\vec{a}_{av} = \frac{\Delta \vec{v}_{inst}}{\Delta t}$

instantaneous acceleration: $\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}_{inst}}{\Delta t}$

So instantaneous acceleration is like finding the average acceleration but over a very small (but not zero) time interval.

Instantaneous velocity/acceleration can be found:
- use calculus (if an equation is known for the position with respect to time)
- use a graph (draw a tangent line at time t)

experimental
- ticker tape
- multi-image photography/video analysis
- motion detector

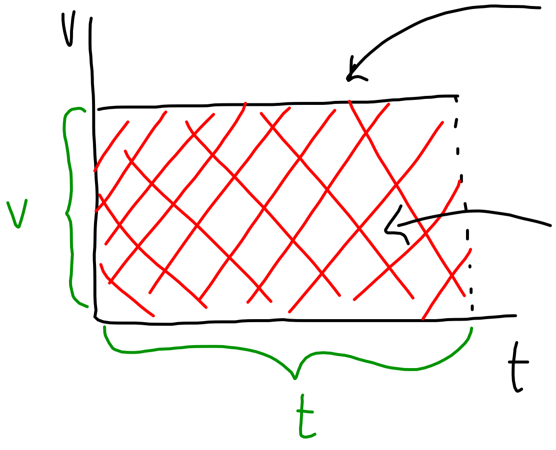
Equations for uniformly accelerated motion

AKA: "The Kinematics equations" or the "suvat" equations

These are a set of 5 equations that can be used to solve kinematics problems involving uniformly accelerated motion

You need to be able to derive the equations. Some are not in the data booklet ⇒ must memorize them.

Consider a velocity - time graph for motion with uniform velocity



slope = 0 (i.e. acceleration is equal to zero (uniform velocity))

area of rectangle = $l \times w$

area of rectangle = vt

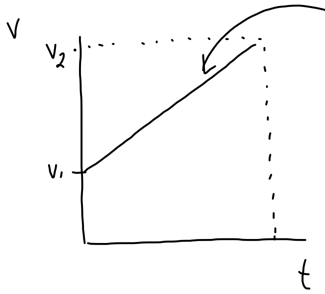
$d = vt$

(area under v-t graph = distance travelled)

① slope (v-t graph) = acceleration

② area under v-t graph = distance

Velocity-Time Graph for Uniformly Accelerated Motion



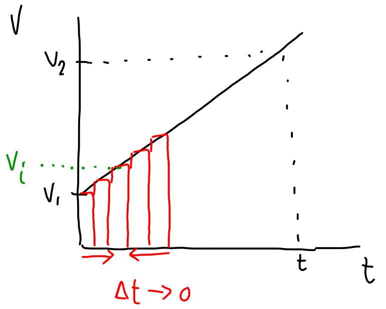
slope = $\frac{\text{rise}}{\text{run}}$

slope = $\frac{\Delta v}{\Delta t}$

$a = \frac{\Delta v}{\Delta t}$

slope (v-t) = acceleration

To determine the distance travelled when the velocity is changing, imagine that the slope of the line is increasing in a large number of small steps.



$$d \approx \sum_{i=1}^{i=N} v_i \, \Delta t$$

height of rectangle ← v_i; width of rectangle ← Δt

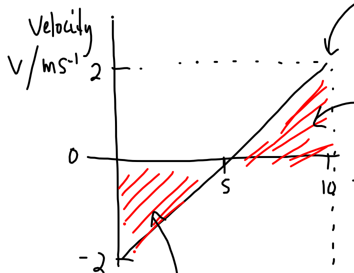
$$d = v_1 \Delta t + v_2 \Delta t + v_3 \Delta t + \ldots + v_N \Delta t$$

We could think about this shape having an area that is equal to the area of an infinite number of rectangles with a width of almost zero.

① slope (v-t) = acceleration

② area (v-t) = distance (or displacement)

Example:



slope = $\frac{2\,ms^{-1}}{5\,s} = 0.4\,ms^{-2}$ ↑ acceleration

$\Delta x\,(\text{area}) = \frac{1}{2}(5s)(2ms^{-1})$

$\boxed{\Delta x = 5\,m}$

↑ area of triangle

$A = \frac{1}{2}bh$

$\Delta x\,(\text{area}) = \frac{1}{2}(5s)(-2ms^{-1})$

$\boxed{\Delta x\,(\text{area}) = -5m}$

overall displacement (over 0 to 10s) = 0

Equation 1

Acceleration: $a = \frac{\Delta V}{\Delta t}$

v is the final velocity

u is the initial velocity.

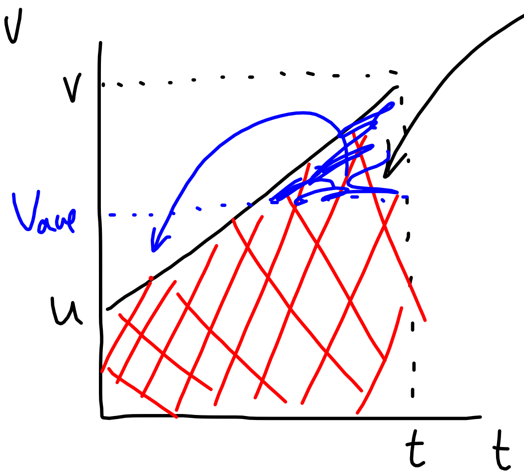
Suvat

$$a = \frac{v - u}{t}$$

$$at = v - u$$

$$\boxed{V = u + at} \quad ①$$

Equation 2



area (v-t) = area of a trapezoid

displacement = $\frac{1}{2}(h_1 + h_2)\, b$

$$s = \frac{1}{2}(u + v)\, t$$

$$s = \underbrace{\left(\frac{u+v}{2}\right)}\, t \quad ②$$

assumed that there is constant acceleration

$$s = V_{ave}\, t$$

Equation 3

Recall Equation 1: $v = u + at$

$v - u = at$ (rearrange for t)

$$t = \frac{v-u}{a}$$

Recall Equation 2: $s = \left(\frac{u+v}{2}\right) t$

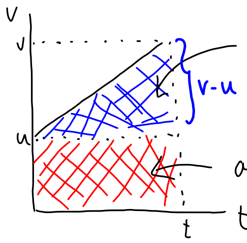
$$s = \left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right)$$

$$s = \frac{v^2 - u^2}{2a}$$

$$2as = v^2 - u^2$$

$$v^2 = u^2 + 2as \quad (3)$$

Equation 4



area of triangle $= \frac{1}{2} bh$

$= \frac{1}{2}(v-u)t$

area of rectangle $= bh$

$= ut$

$$s = ut + \frac{1}{2}(v-u)t$$

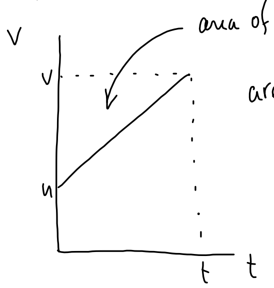
Recall equation 1: $v = u + at$

$v - u = at$

$$s = ut + \frac{1}{2}(at)t$$

$$s = ut + \frac{1}{2}at^2 \quad (4)$$

Equation 5



area of triangle $= \frac{1}{2}(v-u)t$

area of big rectangle $= vt$

$$s = vt - \frac{1}{2}(v-u)t$$

at

$$s = vt - \frac{1}{2}at^2 \quad (5)$$